1 Factoring and Solving Polynomials

1. Completely factor the polynomial.
$$y(x-2)+(x-2)$$

$$\begin{aligned} y(x-2)+(x-2) &= y(x-2)+1(x-2) \\ &= (x-2)(y+1) \end{aligned}$$

2. Completely factor the polynomial.
$$x^3+x^2+x+1$$

   Since there are four terms, try factoring by grouping. The first two terms share a greatest common factor (GCF) of x^2 and the second two terms share the GCF of 1.

$$\begin{aligned} x^3+x^2+x+1 &= x^2(x+1)+1(x+1) \\ &= (x+1)(x^2+1) \end{aligned}$$

3. Completely factor the polynomial.
$$x^3+x^2-x-1$$

   Since there are four terms, try factoring by grouping. The first two terms share a greatest common factor (GCF) of x^2 and the second two terms share the GCF of -1. In the second line of the problem below, notice the difference of squares. You factor this by following the steps or recalling the pattern, but the difference of squares x^2-1 must be factored.

$$\begin{aligned} x^3+x^2-x-1 &= x^2(x+1)-1(x+1) \\ &= (x+1)(x^2-1) \\ &= (x+1)(x+1)(x-1) \end{aligned}$$

4. Completely factor the trinomial.
$$3x^2-60x+108$$

   The first step is to notice the GCF of 3. Then play the game on the quadratic inside the "[ ]".

$$\begin{aligned} 3x^2-60x+108 &= 3[x^2-20x+36] \\ &= 3[x^2-18x-2x+36] \\ &= 3[x(x-18)-2(x-18)] \\ &= 3[(x-18)(x-2)] \\ &= 3(x-18)(x-2) \end{aligned}$$

5. Completely factor the binomial.
$$x^2-16$$

   This binomial looks like a typical quadratic if you recall that the $0x$ term is usually not written.

$$\begin{aligned} x^2-16 &= x^2-4x+4x-16 \\ &= x(x-4)+4(x-4) \\ &= (x-4)(x+4) \end{aligned}$$

   Notice that this problems is also a difference of squares. If you recoginze the pattern, you can utilize the pattern. If not, resort to the factoring steps learned in class.

6. Completely factor the binomial.

$$x^2 + 8$$

This polynomial is prime.

7. Completely factor the binomial.

$$x^3 + 8$$

There is no GCF in the polynomial, and factoring x^3 does not fit our steps for factoring a quadratic. Since the x is cubed, if you can write the 8 as something cubed, then this polynomial is a sum of cubes. We have a formula for factoring such a polynomial.

$$\begin{aligned} x^3 + 8 &= x^3 + 2^3 \\ &= (x+2)(x^2 - 2x + 2^2) \\ &= (x+2)(x^2 - 2x + 4) \end{aligned}$$

8. Completely factor the binomial.

$$x^4 - 16$$

You need to recognize this is a difference of squares. Then notice that a new difference of square is created and also must be factored.

$$\begin{aligned} x^4 - 16 &= (x^2 + 4)(x^2 - 4) \\ &= (x^2 + 4)(x + 2)(x - 2) \end{aligned}$$

9. Completely factor the binomial.

$$2x^3 - 16$$

As with all factoring, check for a GCF, then decide how to factor. In this case 2 is the GCF. Once the GCF is factored, then the factor $x^3 - 8$ is a difference of cubes and can be factored by the formula learned in class.

$$\begin{aligned} 2x^3 - 16 &= 2(x^3 - 8) \\ &= 2(x^3 - 2^3) \\ &= 2(x-2)(x^2 + 2x + 2^2) \\ &= 2(x-2)(x^2 + 2x + 4) \end{aligned}$$

10. Solve the equation.

$$x^2 + 7x = 0$$

Since the equation is already set to zero, follow the factoring steps: Step 1) GCF. In this equation the GCF is x and once factored, the problem is completely factored. There is no need to play the game. Set each factor to zero and solve for x.

$$\begin{aligned} x^2 + 7x &= 0 \\ x(x+7) &= 0 \\ x = 0 \text{ OR } x + 7 &= 0 \\ x = 0 \text{ OR } x &= -7 \end{aligned}$$

11. Solve the equation.

$$x^3 - 12x^2 + 32x = 0$$

This equation is set to zero, so the next step is to factor the GCF of x. Now there are two factors, the second being a quadratic. Factor the quadratic, shown below in the "[ ]", by following the factoring steps learned in class.

$$\begin{aligned}
x^3 - 12x^2 + 32x &= 0 \\
x[x^2 - 12x + 32] &= 0 \\
x[x^2 - 8x - 4x + 32] &= 0 \\
x[x(x-8) - 4(x-8)] &= 0 \\
x[(x-8)(x-4)] &= 0 \\
x(x-8)(x-4) &= 0 \\
x = 0 \text{ OR } x - 8 &= 0 \quad \text{OR} \quad x - 4 = 0 \\
x = 0 \text{ OR } x &= 8 \quad \text{OR} \quad x = 4
\end{aligned}$$

12. Solve the equation.

$$18x^2 + 9x - 2 = 0$$

The equation is set to zero and there is no GCF. Go straight to the game and the following steps.

$$\begin{aligned}
18x^2 + 9x - 2 &= 0 \\
18x^2 + 12x - 3x - 2 &= 0 \\
6x(3x+2) - 1(3x+2) &= 0 \\
(3x+2)(6x-1) &= 0 \\
3x + 2 = 0 \text{ OR } 6x - 1 &= 0 \\
3x = -2 \text{ OR } 6x &= 1 \\
x = \frac{-2}{3} \text{ OR } x &= \frac{1}{6}
\end{aligned}$$

13. Solve the equation.

$$(y-5)(y-2) = 28$$

Multiply the parentheses then set the equation to zero. The problem proceeds similar to the problems above.

$$\begin{aligned}
(y-5)(y-2) &= 28 \\
y^2 - 7y + 10 &= 28 \\
y^2 - 7y - 18 &= 0 \\
y^2 - 9y + 2y - 18 &= 0 \\
y(y-9) + 2(y-9 &= 0 \\
(y-9)(y+2) &= 0 \\
y - 9 = 0 \text{ OR } y + 2 &= 0 \\
y = 9 \text{ OR } y &= -2
\end{aligned}$$

14. Solve the equation.

$$99x^3 + 219x^2 = 120x$$

$$
\begin{aligned}
99x^3 + 219x^2 &= 120x \\
99x^3 + 219x^2 - 120x &= 0 \\
3x(33x^2 + 73x - 40) &= 0 \\
3x[33x^2 - 15x + 88x - 40] &= 0 \\
3x[3x(11x - 5) + 8(11x - 5)] &= 0 \\
3x(11x - 5)(3x + 8) &= 0
\end{aligned}
$$

$$3x = 0 \text{ OR } 11x - 5 = 0 \quad \text{OR} \quad 3x + 8 = 0$$

$$x = 0 \text{ OR } 11x = 5 \quad \text{OR} \quad 3x = -8$$

$$x = 0 \text{ OR } x = \frac{5}{11} \quad \text{OR} \quad x = \frac{-8}{3}$$

15. Completely factor the polynomial. (Hint: *Remember that algebra is the study of patterns.*)

$$x^{32} - 1$$

Factoring this polynomial is completely based on recognizing patterns. The problem is a nesting of differences of squares.

$$
\begin{aligned}
x^{32} - 1 &= (x^{16} + 1)(x^{16} - 1) \\
&= (x^{16} + 1)(x^8 + 1)(x^8 - 1) \\
&= (x^{16} + 1)(x^8 + 1)(x^4 + 1)(x^4 - 1) \\
&= (x^{16} + 1)(x^8 + 1)(x^4 + 1)(x^2 + 1)(x^2 - 1) \\
&= (x^{16} + 1)(x^8 + 1)(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)
\end{aligned}
$$

16. Completely factor the polynomial. (Hint: *Remember that algebra is the study of patterns.*)

$$x^2 - 2$$

This problem is prime if you play the game; however, if you recognize the difference of squares, it can be factored.

$$
\begin{aligned}
x^2 - 2 &= (x)^2 - (\sqrt{2})^2 \\
&= (x + \sqrt{2})(x - \sqrt{2})
\end{aligned}
$$